

Fulde-Ferrell-Larkin-Ovchinnikov - like state in Ferromagnet - Superconductor Proximity System

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We discuss some properties of the ferromagnet - superconductor proximity system. In particular, the emphasis is put on the physics of the Fulde-Ferrell-Larkin-Ovchinnikov (*FFLO*) like state. In addition to Andreev reflections it features a number of unusual thermodynamic and transport properties, like: oscillatory behavior of the pairing amplitude, density of states and superconducting transition temperature as a function of the ferromagnet thickness. Surprisingly, under certain conditions spontaneous spin polarized current is generated in the ground state of such a system. We provide some informations regarding experimental observations of this exotic state.

I. INTRODUCTION

When a normal non-magnetic metal is connected to a superconductor it acquires superconducting properties, like non-zero pairing amplitude. This effect, known as the proximity effect [1], has extensively been studied for almost half a century. It is rather well understood by now in terms of Andreev reflections [2], according to which an impinging electron (with energy less than superconducting gap) on the normal metal (*NM*) / superconductor (*SC*) interface is reflected back as a hole and the Cooper pair is created in superconductor. From the point of view of Andreev reflections the proximity effect can be regarded as a non-zero density of the Andreev correlated electron - hole pairs on the normal metal side of the interface.

When a normal metal is replaced by a ferromagnet (FM), another energy scale enters problem, namely the exchange splitting which is related to the spin polarization of the electrons. Such FM/SC hybrid structures are important from the scientific point of view, as they allow the study of the interplay between ferromagnetism and superconductivity [3] as well as of device applications in such areas of technology as magnetoelectronics [4] or quantum computing [5].

It is widely accepted that ferromagnetism and superconductivity are two antagonistic phenomena, so one could expect that the proximity effect in FM/SC system should be suppressed. Indeed, the one can argue that in ferromagnet there are different numbers of spin-up (majority) n_{\uparrow} and spin-down (minority) n_{\downarrow} conduction channels, and due to the fact that incident and reflected particles occupy different spin bands, only a fraction $n_{\downarrow}/n_{\uparrow}$ of majority particles can be Andreev reflected [6].

On the other hand if an exchange field acts on the Cooper pairs, one would expect that either it is too weak to break the pair, or it suppresses completely superconductivity. However when a Cooper pair is subjected to the exchange field, it acquires a finite momentum and for certain values of the exchange splitting a new superconducting state is realized, known as *Fulde - Ferrell - Larkin - Ovchinnikov (FFLO)* state [7, 8]. Interestingly such state features a spatially dependent order parameter corresponding to the non-zero center of mass motion of the Cooper pairs. This state features in non-zero spin polarization, almost normal tunneling characteristics and almost normal Sommerfeld specific heat ratio, anisotropic electrodynamic properties. Unfortunately the bulk state is very sensitive to the impurities and shape of the Fermi surface. Another novel feature of this state is a current flowing in the ground state. The unpaired electrons tend to congregate at one portion of the Fermi surface so a quasiparticle current is produced. In order to satisfy the Bloch theorem: no current in the ground state, a supercurrent, generated by the nonzero value of the pairing momentum, flows in opposite direction, and the total current is zero.

Similar oscillations of the pairing amplitude have been predicted [9]-[12] in ferromagnet/superconductor proximity systems. It turns out that these oscillations are responsible for the oscillatory behavior of the SC critical temperature T_c , first experimentally observed by Wong *et al.* [13], and the density of states [14] as the thickness of the FM slab is varied. In fact, the oscillations of the T_c in FM/SC multilayers can be also explained in terms of the effective π -junction behavior [10]. It was shown that at specific FM thickness the Josephson

coupling between two SC layers can lead to a junction with an intrinsic phase (of the order parameter) difference $\delta\varphi = \pi$, which exhibits a higher T_c than the ordinary one ($\delta\varphi = 0$). The π -junction effect has been originally proposed by Bulaevskii *et al.* [15] to arise in the tunnel barriers containing magnetic impurities. It was also suggested that the π -junction can be realized in high- T_c superconducting weak links [16], where the SC order parameter changes its sign under $\pi/2$ rotation. This has tremendous consequences as it leads to many important effects [17, 18], like: the zero energy Andreev states, zero-bias conductance peaks, large Josephson current, time reversal symmetry breaking, paramagnetic Meissner effect and spontaneously generated currents.

From the point of view of the present paper the important issue is the formation of the Andreev bound states in FM/SC proximity system. The Andreev states arise due to the fact that the quasiparticles of the ferromagnet participating in the Andreev reflections move along closed orbits. Such states have been first studied by de Gennes and Saint-James [19] in the insulator/normal metal/superconductor ($I/NM/SC$) trilayer. The energies of these states are always smaller than the SC gap Δ and symmetrically positioned around the Fermi level. They strongly depend on the geometry of the system as well as on the properties of the interfaces. In high- T_c (d -wave) superconductors, these states can be shifted to zero energy, due to the specific form of the symmetry of the order parameter [20], thus indicating π -junction behavior in the system. Naturally, such Andreev states can also arise in the $I/FM/SC$ heterostructures. Moreover, it is possible to shift the energies of these states by changing the exchange splitting, as was first demonstrated by Kuplevakhskii & Fal'ko [21]. In turn, by properly adjusting the exchange splitting the position of the Andreev bound states can be moved to the Fermi energy. The system under such circumstances behaves like that being in the π -junction phase as the spontaneous current is generated [22].

Some of our results have already been published [22]-[24]. Here we wish to present a more detailed study of the FM/SC proximity system in terms of $FFLO$ physics. In some situations the ground state of FM/SC structures has properties of both the $FFLO$ and the π -junction, leading to various interesting and unexpected phenomena.

The paper is organized as follows: In Sec. II the simple model which allows for self-consistent description of the FM/SC heterostructure is introduced. In Sec. III the nature of the Andreev bound states in the ferromagnet is discussed. The spontaneously generated current and corresponding magnetic field in the ground state are studied in the Sec. IV. In

Sec. V show some transport properties of the system, in the Sec. VI we compare our system to usual *FFLO* state, and finally, we conclude in Sec. VI.

II. MODEL AND THEORY

To study the properties of *FM/SC* system we have adopted the 2D Hubbard model featuring the exchange splitting in the ferromagnet and an electron - electron attraction in superconductor. The Hamiltonian is:

$$H = \sum_{ij\sigma} \left[t_{ij} + \left(\frac{1}{2} E_{ex} \sigma - \mu \right) \delta_{ij} \right] c_{i\sigma}^+ c_{j\sigma} + \frac{1}{2} \sum_{i\sigma} U_i n_{i\sigma} n_{i-\sigma} \quad (1)$$

where in the presence of a vector potential $\vec{A}(\vec{r})$, the hopping integral is given by $t_{ij} = -t e^{-ie \int_{\vec{r}_i}^{\vec{r}_j} \vec{A}(\vec{r}) \cdot d\vec{r}}$ for nearest neighbor lattice sites, whose positions are \vec{r}_i and \vec{r}_j , and zero otherwise. The exchange splitting E_{ex} is only non-zero on the *FM* side, unlike as U_i (electron - electron attraction) being non-zero only in *SC*. μ is the chemical potential, $c_{i\sigma}^+$, ($c_{i\sigma}$) are the usual electron creation (annihilation) operators and $\hat{n}_{i\sigma} = c_{i\sigma}^+ c_{i\sigma}$.

In the following we shall work within Spin - Polarized - Hartree - Fock - Gorkov (*SPHFG*) approximation [22] assuming periodicity in the direction parallel to the interface while working in a real space in the direction perpendicular. Labeling the layers by integer n and m at each k_y point of the Brillouin zone we shall solve the following *SPHFG* equation:

$$\sum_{m', \gamma, k_y} H_{nm'}^{\alpha\gamma}(\omega, k_y) G_{m'm}^{\gamma\beta}(\omega, k_y) = \delta_{nm} \delta_{\alpha\beta} \quad (2)$$

where the only non-zero elements are: H_{nm}^{11} and $H_{nm}^{22} = (\omega - \frac{1}{2} \sigma E_{ex} \pm \mu \pm t \cos(k_y \mp eA(n))) \delta_{nm} \pm t \delta_{n,n+1}$ for the upper and lower sign respectively, $H_{nm}^{33} = H_{nm}^{11}$ and $H_{nm}^{44} = H_{nm}^{22}$ with σ replaced by $-\sigma$ and $H_{nm}^{12} = H_{nm}^{21} = -H_{nm}^{34} = -H_{nm}^{43} = \Delta_n \delta_{nm}$ and $G_{nm}^{\alpha\beta}$ is corresponding Green's function (*GF*).

As usual, the self-consistency is assured by the relations determining the *FM* (m_n) and *SC* (Δ_n) order parameters, current ($J_{y\uparrow(\downarrow)}(n)$) and the vector potential ($A_y(n)$) respectively:

$$m_n = n_{n\uparrow} - n_{n\downarrow} = \frac{2}{\beta} \sum_{k_y} \sum_{\nu=0}^{2N-1} \text{Re} \left\{ (G_{nn}^{11}(\omega_\nu, k_y) - G_{nn}^{33}(\omega_\nu, k_y)) e^{(2\nu+1)\pi i/2N} \right\} \quad (3)$$

$$\Delta_n = U_n \sum_{k_y} \langle c_{n\downarrow}(k_y) c_{n\uparrow}(k_y) \rangle = \frac{2U_n}{\beta} \sum_{k_y} \sum_{\nu=0}^{2N-1} \text{Re} \left\{ G_{nn}^{12}(\omega_\nu, k_y) e^{(2\nu+1)\pi i/2N} \right\} \quad (4)$$

$$J_{y\uparrow(1)}(n) = \frac{4et}{\beta} \sum_{k_y} \sin(k_y - eA_y(n)) \sum_{\nu=0}^{2N-1} \text{Re} \left\{ G_{nn}^{11(33)}(\omega_\nu, k_y) e^{(2\nu+1)\pi i/2N} \right\} \quad (5)$$

$$A_y(n+1) - 2A_y(n) + A_y(n-1) = -4\pi J_y(n) \quad (6)$$

The details of the calculations can be found in [23].

III. ANDREEV BOUND STATES

Before we discuss results of fully self-consistent calculations we would like to turn the attention to origin of Andreev bound states and take a look at physics of them from the point of view of semiclassical approach.

From quasiclassical considerations, each bound state corresponds to quasiparticle moving along a family of closed trajectories [25]. The energy of such bound state is determined by the Bohr-Sommerfeld quantization rules, according to which the total phase accumulated during one cycle has to be equal to multiples of 2π . Interestingly, the bound states also emerge in the normal metal/superconductor (NM/SC) structures [19] due to the Andreev reflections [2], according to which an incident electron is reflected back as a hole at the interface, and a Cooper pair is created in SC . Such states are built up from a combination of electron and hole wave functions. The example of the closed quasiparticle trajectory, producing the bound state, in an insulator/(normal metal)/superconductor $I/NM/SC$, is shown in the Fig. 1. It consists of an electron e segment, which includes a ordinary reflection at the I/NM interface, and hole h one, retracing backwards the electron trajectory. The total accumulated phase in this case consists of contribution from Andreev reflections at point A : $-\alpha_1 + \varphi_1$ and B : $-\alpha_2 + \varphi_2$ as well as contribution from the propagation through the normal metal $\beta(E)$. $\alpha_{1(2)} = \arccos(E/|\Delta_0|)$ is the Andreev reflection phase shift, while $\varphi_{1(2)}$ is the phase of the SC order parameter at point A (B). $\beta(E) = 2L(k_e - k_h) + \beta_0$ is the electron-hole dephasing factor and describes the phase acquired during the propagation through the normal region, where the first term corresponds to the ballistic motion and the second one to the reflection at the I/NM surface. L is the thickness of NM , and k_e (k_h) is the electron (hole) wave vector. Thus the Bohr-Sommerfeld quantization condition is:

$$-(\alpha_1 + \alpha_2) \pm (\varphi_1 - \varphi_2) + \beta(E) = 2n\pi \quad (7)$$

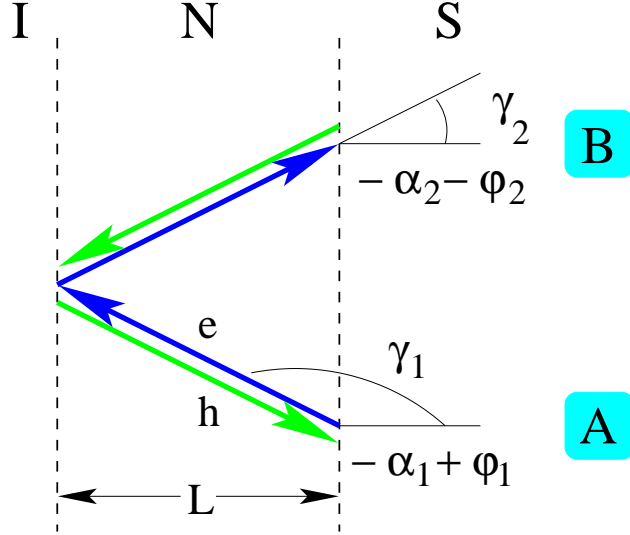


FIG. 1: The example of the quasiparticle path corresponding to the Andreev reflections, giving a bound state. The quasiparticle is trapped in the normal region because of normal reflection at the I/NM surface and the Andreev reflection at the NM/SC interface. The total phase accumulated during one cycle is equal: $-(\alpha_1 + \alpha_2) \pm (\varphi_1 - \varphi_2) + \beta(E)$.

where the $\pm(\varphi_1 - \varphi_2)$ stands for the trajectories in the $\pm k_y$ (parallel to the interface) direction.

If there is no phase difference between points A and B in the Fig. 1 (for example NM/SC interface), the bound states always appear in pairs symmetrically positioned around the Fermi level because of the time reversal symmetry in the problem. Moreover, due to the fact that there is no difference between electrons and holes at the Fermi level ($\beta(E = 0) = 0$), there is no $E = 0$ solution. In other words, the bound states always emerge at finite energies.

The situation is quite different if there is a phase difference $(\varphi_1 - \varphi_2)$ between points A and B (see Fig. 1). The example can be the interfaces with d -wave superconductors oriented in the (110) direction, where $(\varphi_1 - \varphi_2) = \pi$. In this case, due to the additional phase shift π the bound states can emerge even at zero energy. Such zero-energy Andreev bound states, in the case of high- T_c superconductors, have been predicted by Hu [20] and are known as *zero-energy mid-gap states*. The presence of the Andreev bound states at zero energy features in many important effects, like zero-bias conductance peaks, π -junction behavior, anomalous temperature dependence of the critical Josephson current, paramagnetic Meissner effect,

time reversal symmetry breaking and spontaneous interface currents [17, 18].

Although the zero-energy states (*ZES*) are likely to appear when the phase of the order parameter at the interface is not constant, the resulting density of states at the Fermi energy is energetically unfavorable and any mechanism able to split these states will lower the energy of the system [18, 26]. One of these is the self-induced Doppler shift [17, 27] $\delta = ev_F A$, where A is a vector potential. The situation is schematically depicted in the Fig. 2. At low

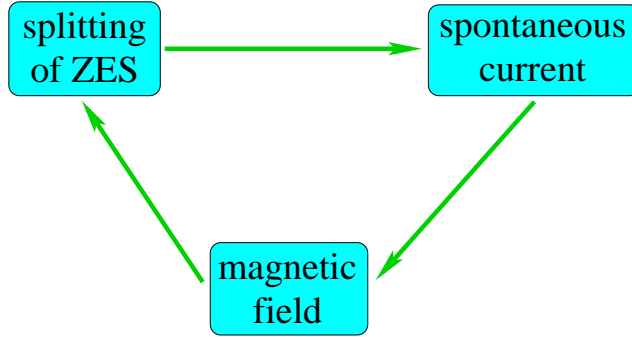


FIG. 2: Generating of the spontaneous currents.

temperature ($T^* \approx (\xi_0/\lambda)T_c$, where λ is the penetration depth of the magnetic field) the splitting of the zero energy states produces a surface current. This current generates a magnetic field (screened by a supercurrent), which further splits *ZES* due to the Doppler shift effect. The effect saturates when the magnetic energy is equal to the energy of the Doppler shifted *ZES*.

Naturally, the Andreev bound states also arises in *I/FM/SC* heterostructures [21, 23, 24, 28, 29, 30, 31, 32]. More importantly, as it was first predicted by Kuplevakhskii & Fal'ko [21], it is possible to shift these states to zero energy by tuning the exchange splitting. So the crossing of the zero energy solution can be obtained either by changing the phase difference ($\varphi_1 - \varphi_2$) or by varying *FM* coherence length (exchange field).

The properties of such bound states have been also studied fully quantum-mechanically within lattice models of the *FM/SC* systems [23, 24, 31] and similar their behavior have been obtained. Interestingly, it turns out, that as in the case of the high- T_c structures [27], such zero energy Andreev states support spontaneous currents flowing in the ground state of the *FM/SC* system [22, 23, 24]. The mechanism of generating of such currents is the same, as earlier discussed, namely the self-induced Doppler shift. So in fact, when

the current flows, such one of the states will be twice shifted: once due to the exchange (Zeeman) splitting, and the second time due to the Doppler shift.

For energies less than superconducting gap, the only Andreev bound states will contribute to the density of states $\rho(E)$. However, as it was mentioned, for fixed thickness and exchange splitting, there will be Andreev bound states at different energies, for different angles of particle incidence (γ_2 in the Fig. 1). Thus to get the density of states, one has to sum the energies of these states over all values of γ_2 :

$$\rho(E) = \sum_{\gamma_2=-\pi/2}^{\pi/2} \delta(E - E_{bound}) \quad (8)$$

and talk, in fact, about Andreev bands rather than single states. However, all that was said on properties of the bound states, remains true for Andreev bands too. In particular the splitting of the whole band due to the spontaneous current is illustrated in the Fig. 3. The additional structure comes from the other (higher order) Andreev reflections.

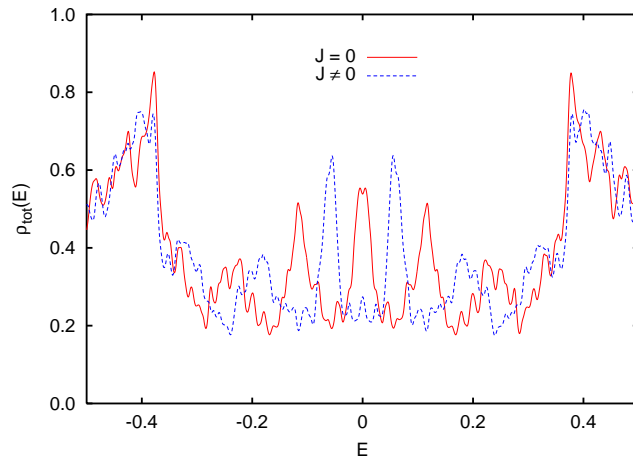


FIG. 3: Doppler splitting of the zero-energy state. From Ref. [23].

Superconducting energy gap $\Delta_0 = 0.376$ in this figure.

There is also a strong correlation between Andreev bound states (bands) and the pairing amplitude [22, 23, 31]. Each time the pairing amplitude at the I/FM interface changes its sign, the Andreev bound state (band) crosses the Fermi energy. Moreover in this case the spontaneous current is generated.

From the experimental point of view the density of states, in particular its temperature dependence, can be a good measure of the current carrying ground state. At certain thicknesses of FM for which the current flows there is a huge drop in the $\rho_{tot}(\varepsilon_F)$ at characteristic

temperature $T^* \approx (\xi_S/\lambda)T_c$, where ξ_S and λ are coherence length and penetration depth respectively. T^* simply indicates the temperature at which magnetic instability, which leads to the generation of the current, takes the place. Such behavior is depicted in the Fig. 4 and should be observable experimentally. If there is no current the *DOS* is due to the Andreev band and is almost constant (we are well below T_c), and as soon as the current starts to flow the Andreev band splits so we observe a drop in $\rho_{tot}(\varepsilon_F)$. The important point is that T^* and T_c are different temperatures.

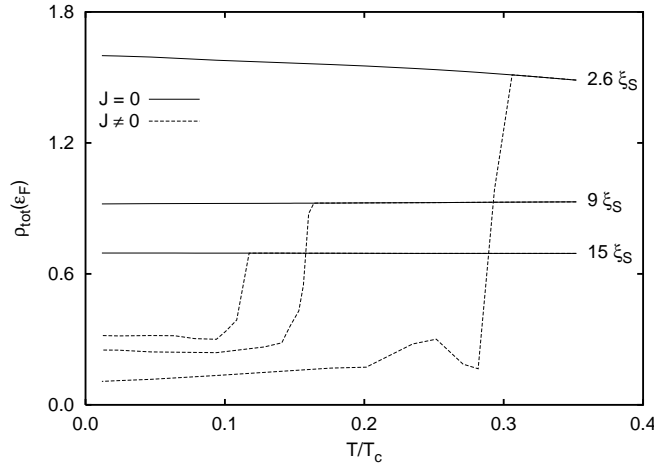


FIG. 4: The temperature dependence of the surface (*FM*/vacuum) density of states at the Fermi energy for various thicknesses of the *FM* slab in the figure. The solid (dashed) line corresponds to the solution without (with) the current. From Ref. [24]

IV. SPONTANEOUS CURRENT

The most remarkable feature of our calculations is that the solution of the *SPHFG* equations frequently converges to a solution with the finite current $j_y(n)$ even though the external vector potential is zero. The typical example of such a current, flowing parallel to the *FM*/*SC* interface, ($j_y^{tot}(n) = j_{y\uparrow}(n) + j_{y\downarrow}(n)$) is shown in the Fig. 5 for a few values of the exchange splitting. Behavior of the current, as a function of the layer index, is very similar to the density of states at the Fermi level. The oscillating nature of the current comes from the Friedel like oscillations of the *DOS* [23]. This is because current is proportional to the *DOS* at the Fermi level. Within semiclassical calculations, which neglect these effects the current is very smooth [32].

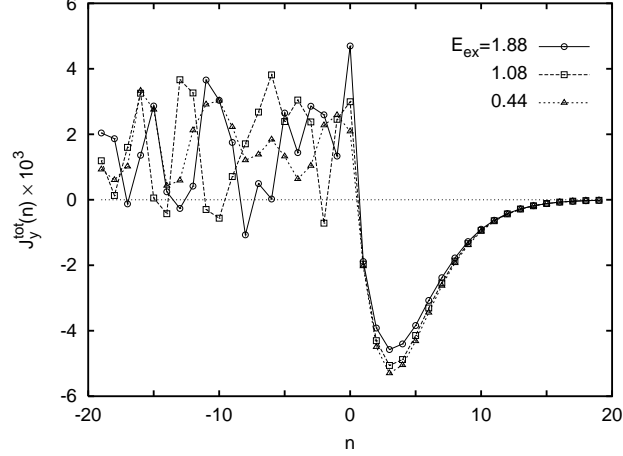


FIG. 5: The total (spontaneous) current $j_y^{tot}(n) = j_{y\uparrow}(n) + j_{y\downarrow}(n)$ flowing parallel to the *FM/SC* interface for a number of exchange splittings. From Ref. [23].

Another important issue is the distribution of the current through the whole trilayer structure. We find that it flows mostly in the positive y direction on ferromagnetic side and in the negative direction in the superconductor. Notably the total current, integrated over the whole sample, is equal to zero within numerical accuracy. This is as it should be for the true ground state and found to be in the *FFLO* state, where the current associated with the unpaired electrons is balanced by the supercurrent flowing in the opposite direction. Similarly here (see Fig. 6).

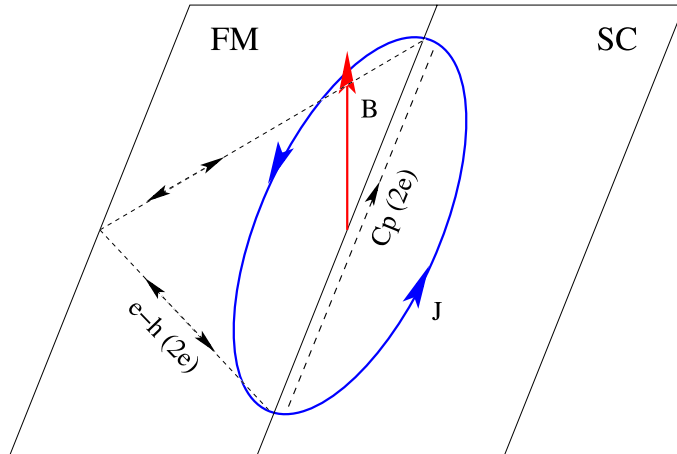


FIG. 6: Schematic view of the current distribution.

Obviously, the spontaneous current distribution (see Fig. 6) generates the magnetic field through the sample. The total magnetic flux weakly depends on the thickness of the sample

and the exchange splitting. Its magnitude is found to be a fraction of the flux quantum $\Phi_0 = h/2e$ and is smaller than upper critical field of the bulk superconductor. This is rather a large field and could be observable in temperature dependent measurements (see Fig. 7).

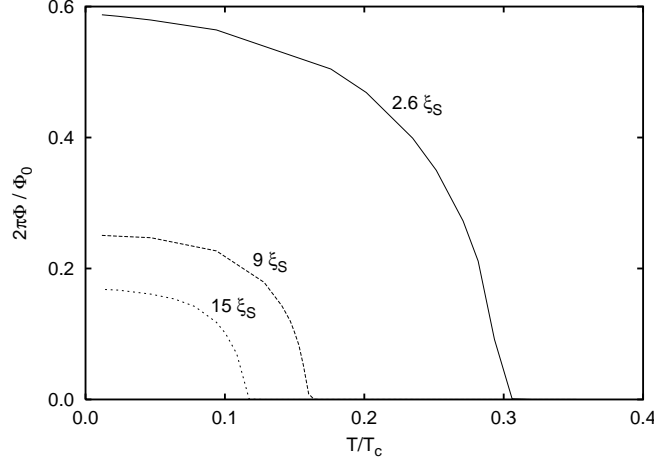


FIG. 7: The temperature dependence of the total magnetic flux for thickness of the *FM* slab $L/\xi_S = 2.6$ (solid), 6 (dashed) and 15 (dotted curve).

V. TRANSPORT PROPERTIES

Some information on spontaneous currents can be also obtained from conductance calculations. To do so we attached a normal metal electrode to our *FM/SC* system and calculate current through *NM/FM/SC* system using nonequilibrium Keldysh Green's function technique [33]. To calculate this current in terms of various physical processes we went along the way outlined in Ref. [34] and got corresponding spin polarized formula for the current as a sum of four different contributions $I = I_1 + I_2 + I_3 + I_A$, where:

$$I_1 = 4\pi^2 t_{NF}^2 \frac{e}{\hbar} \sum_{\sigma} \int d\omega |1 + G_{FN\sigma}^{11r}(\omega)|^2 \rho_{NN\sigma}^{11}(\omega) \rho_{FF\sigma}^{11}(\omega) [f(\omega - eV) - f(\omega)] \quad (9)$$

$$I_2 = 8\pi^2 t_{NF}^2 \frac{e}{\hbar} \sum_{\sigma} \int d\omega \text{Re}\{t_{NF} G_{NF\sigma}^{21a}(\omega) [1 + G_{FN\sigma}^{11r}(\omega)]\} \rho_{NN\sigma}^{11}(\omega) \rho_{FF\sigma}^{12}(\omega) [f(\omega) - f(\omega - eV)] \quad (10)$$

$$I_3 = 4\pi^2 t_{NF}^4 \frac{e}{\hbar} \sum_{\sigma} \int d\omega |G_{FN\sigma}^{12}(\omega)|^2 \rho_{NN\sigma}^{11}(\omega) \rho_{FF-\sigma}^{22}(\omega) [f(\omega - eV) - f(\omega)] \quad (11)$$

$$I_A = 4\pi^2 t_{NF}^4 \frac{e}{\hbar} \sum_{\sigma} \int d\omega |G_{FF\sigma}^{12}(\omega)|^2 \rho_{NN\sigma}^{11}(\omega) \rho_{LL-\sigma}^{22}(\omega) [f(\omega - eV) - f(\omega + eV)] \quad (12)$$

I_1 corresponds to normal electron tunneling between electrodes, I_2 is a net transfer of single electron with creation or annihilation of pairs as an intermediate state. I_3 corresponds to a process in which electron from normal electron is converted to a hole in superconductor - branch crossing process in language of *BTK* theory [35], while I_A is the Andreev tunneling.

The differential conductance $G(eV) = dI/d(eV)$ as a function of $eV = \mu_{NM} - \mu_{SC}$ is shown in the Fig. 8. Clearly, if there is a spontaneous current in the ground state, the

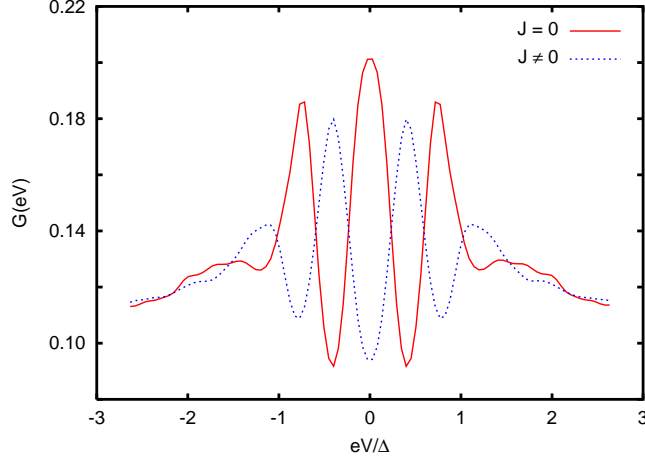


FIG. 8: The total differential conductance for the solution with and without the spontaneous current.

conductance peak is split, similarly as in the *DOS*. We could expect such behavior because $G(eV)$ is proportional to the *DOS* at the Fermi energy. And again this effect could be observable in the tunneling experiments.

We have also extracted Andreev conductance from the total one and plotted in the Fig. 9. We can see that conductance associated with the Andreev processes is strongly enhanced when the current flows in the ground state. Unfortunately it could be very difficult experimentally measure Andreev conductance only. Despite the fact that for energies less than *SC* gap the only allowed process is Andreev reflection, as in the point contact geometry, it doesn't work in our system. The problem is that even at very low energies there is a finite *DOS* at the Fermi level due to ferromagnet. Naturally the pairing amplitude is induced in *FM* slab but this is not true energy gap in the quasiparticle spectra and we always deal with some single electron processes in tunneling events.

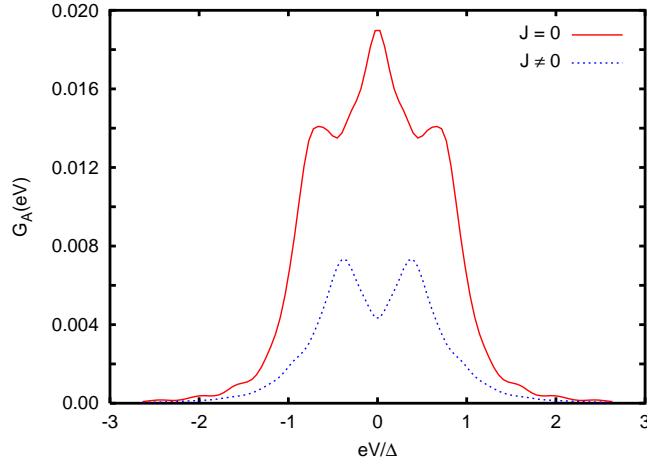


FIG. 9: Corresponding Andreev differential conductance for the solution with and without the spontaneous current.

VI. 2D FFLO STATE

Before closing discussion on the spontaneous current we wish to make a remark regarding the nature of the ground state in our system. To begin with we recall that recently it has been predicted [36] that under certain conditions a *3D-FFLO* state is energetically more favorable than usual *1D* state. The *3D* state manifests itself in oscillatory behavior of the pairing amplitude not only in the direction perpendicular to the interface but also in direction parallel to it. Moreover, changing the thickness of the *FM* slab, one can switch the ground state of the system between *3D* and *1D-FFLO* state [23, 36].

The current carrying ground state of our system can be interpreted as a *2D-FFLO* state. The argument is as follows: The oscillations of the pairing amplitude in the direction perpendicular to the interface occur regardless whether the spontaneous current flows or not. Within the *FFLO* theory [7, 8], the period of the oscillations is related to the *x*-component of the center of mass momentum of the Cooper pair \mathbf{Q} . On the *FM* side of our model the *FFLO* periodicity is governed by $\mathbf{Q} = (2E_{ex}/v_F) \frac{\mathbf{v}_F}{v_F}$, where \mathbf{v}_F is the Fermi velocity vector. This can be interpreted as the usual *1D-FFLO* state in confined geometry. On the other hand, when the current flows parallel to the interface, there is a finite vector potential in the *y*-direction. This can be regarded as a *y*-component of the \mathbf{Q} -vector. So one can say that when the spontaneous current flows, the *2D-FFLO* state is realized. Moreover when the *FM* thickness is changed the ground state of the system is switched between *2D*- and *1D*-

state, which manifests itself in spontaneous current flow or in the lack of it. In the present calculations this vector was found during the self-consistency procedure, as it is related to the vector potential in the y -direction. Moreover, the effective Q_y changes its value from layer to layer leading to inhomogeneous *FFLO*-like state in both dimensions.

VII. CONCLUSIONS

The competition between ferromagnetism and superconductivity in *FM/SC* heterostructures give rise to the Fulde - Ferrell - Larkin - Ovchinnikov (*FFLO*) state in these systems. The original bulk *FFLO* state manifests itself in a spatial oscillations of the *SC* order parameter as well as in spontaneously generated currents flowing in the ground state of the system. We have argued that a very interesting version of this phenomenon accrues in *FM/SC* proximity systems. In short, due to the proximity effect and Andreev reflections at the *FM/SC* interface, the Andreev bound states appear in the quasiparticle spectrum. These states can be shifted to the zero energy by tuning the exchange splitting or the thickness of the ferromagnet, thus they became zero-energy mid-gap states which lead to various interesting effects. In particular, the occurrence of spontaneous currents in the ground state can be related to the zero-energy states, as in the case of high- T_c superconductors. It seems that some combination of both phenomena is realized in a real systems. The fact that oscillatory behavior of *SC* order parameter is strongly correlated with the crossing of the Andreev bound states through Fermi energy and the generation of the spontaneous currents further support *FFLO* - Andreev bound states picture. The experimental confirmation of the existence of the spontaneous (spin polarized) currents in the ground state would support the *FFLO* - Andreev bound states scenario in these structures.

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